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Research article

MATHEMATICAL MODELING AND SIMULATION TO PREDICT THE DEFORMATION RATE FROM MAGNESIUM IN CONCRETE PAVEMENTS

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Abstract

The rate of magnesium compound depositing in concrete formation has been noted to be one of the substances that deform the compressive strength of concrete pavement. Magnesium deposition in rigid pavement are destructive to the constituents by replacing the calcium in hydrated Portland cement since it is highly soluble, it generates more concentration than sodium sulfate. It has been noted that strong solution of magnesium sulfate are capable of reacting with other calcium silicate hydrate phase as well as hydroxide and calcium aluminate hydrate phases. The concentration of the magnesium may definitely affect the course of the reaction in concrete pavement. These deformations has developed pavement failure as design lifespan of concrete pavement formation reduce, failing before its expected design lifespan. Mathematical model were developed to monitor the rate of deformation from magnesium sulfate in concrete pavements. The derived model solutions were simulated, and it produced theoretical values compared with the experimental values that determine the rate of deformation from magnesium sulfate in concrete pavements. These parameters developed a best fit that established validation of the model. Experts will definitely apply this conceptual formation favuorable in monitoring the concentration including application of the model it in design of concrete pavements thus attaining its design lifespan.

Keywords: Modeling and Simulation, Deformation Rate, Magnesium and Concrete Pavements.

1. Introduction

The moving of chloride ions into concrete is a complex process which involves diffusion, capillary suction, penetration and convective flow through the pore system and microcracking network, accompanied by physical adsorption and chemical binding. With such a complex moving process, it is necessary to understand individual transport mechanisms and the predominant transport process in order to pinpoint the appropriate method for quantifying the chloride resistance of concrete. Concrete provides physical and chemical protection to the reinforcing steel from penetrating chlorides which may cause steel depassivation leading to increased risk of steel corrosion. The chloride resistance depends on the permeability of the concrete and the thickness cover of the reinforcement. The integrity of the concrete cover under service load, in terms of cracking and crack width, also influences the resistance to penetrating chlorides. Corrosion of steel reinforcement is an electrochemical process. Hence electrochemical properties of concrete, such as resistivity, are important inherent properties affecting the corrosion rate of reinforcing steel. More so, The physical and mechanical properties of pervious concretes are reported elsewhere (Onstenk et al. 1993, Yang and Jiang 2003, Neithalath 2004, Neithalath et al. 2005, 2006, Tennis et al. 2004). The use of larger aggregate sizes up to 20 mm maximum size has been recommended for pervious concrete since they result in large sized pores in the material as well as reduced clogging (Nelson and Phillips 1994). Recently, ACI Committee 522 has suggested that the aggregate sizes for pervious concrete should be between 9.5 mm and 19 mm, and that no fine aggregates should be used. The water permeation capacity or the drainage properties are closely related to the accessible porosity. For an accessible porosity of 20-29%, the coefficient of permeability is about 0.01 m/s (Belgian Road Research Center Report BE 3415, 1994). It has also been reported that the fine aggregate content determines the permeability. A drainage rate of 100 to 750 l/min/m2 has been reported for several pervious concretes (Tennis et al. 2004). Intrinsic permeability of 1 x 10-10 m2 to 5 x 10-10 m2 has been reported for pervious concretes with porosity ranging from 17% to 28% (Neithalath et al. 2006). The mixture proportions used in this study are given. This is adopted from a larger study directed towards ascertaining the influence of material structure of pervious concrete on hydraulic conductivity and acoustic absorption (Neithalath 2004, Marolf et al. 2004).

2. Governing Equation

$$\theta_w V \frac{\partial c}{\partial t} = \theta_M \frac{\rho_B}{\rho_W} V \frac{\partial^2 c}{\partial x^2} \qquad \text{……………………………….} \quad (1)$$

Substituting solution C = ZT into (1), we have

$$\theta_w vZT^1 = \vartheta \frac{\rho_b}{\rho_w} VZ^{11}T \qquad \text{……………………………….} \quad (2)$$

$$\vartheta_w \frac{T^1}{T} = -\theta_M \frac{\rho_b}{\rho_w} V \frac{Z^{11}}{Z} T \qquad \text{……………………………} \quad (3)$$

$$\theta_W \frac{T^1}{T} \quad -\vartheta \frac{\rho_b}{\rho_w} V\left(\frac{Z^{11}}{Z}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (4)$$

$$\vartheta_W V \frac{T^1}{T} \quad -\frac{Z^{11}}{Z} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (5)$$

Considering when Ln x 0 →

$$\theta_W V \frac{T^1}{T} \quad = \vartheta_M \frac{\rho_b}{\rho_W} V \frac{Z^{11}}{Z} - T = \lambda^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (6)$$

$$\theta_W V \frac{T^1}{T} \quad = \lambda^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (7)$$

$$\frac{Z^{11}}{Z} \quad = \lambda^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \qquad (8)$$

$$\theta_M \frac{\rho_b}{\rho_W} V \quad = \lambda^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \qquad (9)$$

This implies that equation (10) can be expressed as:

$$\theta_M \frac{\rho_b}{\rho_w} v \frac{Z^{11}}{Z} \quad = \lambda^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \qquad (10)$$

$$\theta_M \frac{\rho_b}{\rho_w} V \frac{Z^2}{Z} \quad = \lambda^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \qquad (11)$$

$$\vartheta_W V \frac{dy^2}{dx^2} \quad = \lambda^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \qquad (12)$$

$$\theta \frac{\rho_b}{\rho_w} V \frac{dy^2}{dz^2} \quad = \lambda^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \qquad (13)$$

$$\vartheta_W V \frac{d^2 y}{dz^2} \quad = \lambda^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \qquad (14)$$

$$\frac{d^2 y}{dz^2} \quad = \frac{\lambda^2}{\vartheta_W V} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \qquad (15)$$

$$d^2 y = \left(\frac{\lambda^2}{\theta_W V} \right) dz^2 \qquad \text{……………………………….. (16)}$$

$$\int d^2 y = \int \frac{\lambda^2}{\vartheta_W V} dz^2 \qquad \text{……………………………….. (17)}$$

$$dy = \frac{\lambda^2}{\vartheta_W V} x\ dz \qquad \text{……………………………….. (18)}$$

$$\int dy = \int \frac{\lambda^2}{\vartheta_W V} Z\ dz + C_1 \qquad \text{……………………………….. (19)}$$

$$y = \frac{\lambda^2}{\theta_w V} Z + C_1 x + C_2 \qquad \text{……………………………….. (20)}$$

$$y = \frac{\lambda^2}{\theta_W V} + C_1 x + C_2 \qquad \text{……………………………….. (21)}$$

$$y = 0$$

$$\Rightarrow \frac{\lambda^2}{\theta_W V} X^2 + C_1 x + C_2 = 0 \qquad \text{……………………………….. (22)}$$

Applying quadratic expression, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{……………………………….. (23)}$$

Where $a = \frac{\lambda^2}{\theta_W V}$, $b = C_1$ and $c = C_2$

$$X = \frac{-(C_1) \pm \sqrt{(C)^2 - 4\left(\frac{\lambda^2}{\theta_W V} \right) C_2}}{2 \frac{\lambda^2}{\theta_W V}} \qquad \text{……………………… (24)}$$

$$= \frac{-C_1 \pm \sqrt{C_1^{\,2} - 4C_2 \frac{\lambda^2}{\theta_W V}}}{2\frac{\lambda^2}{\theta_W V}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (25)$$

$$X = \frac{-C_1 + \sqrt{C_1^{\,2} - 4C_2 \frac{\lambda^2}{\theta_W V}}}{2\frac{\lambda^2}{\theta_W V}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (26)$$

$$X = \frac{-C_1 + \sqrt{C_1^{\,2} - 4C_2 \frac{\lambda^2}{\theta_W V}}}{2\frac{\lambda^2}{\vartheta_W V}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (27)$$

$$X = \frac{-C - \sqrt{C_1^{\,2} - 4C_2 \frac{\lambda^2}{\vartheta_W V}}}{2\frac{\lambda^2}{\vartheta_W V}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (28)$$

Substituting equation (20) to the following boundary conditions and initial values condition

$$t = 0 \quad C = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (29)$$

Therefore, $X_{(x)} = C_1 \ell^{-mx} + C_2 \ell^{m_2 x}$ .... (30)

$$C_1 \, Cos \, M_1 x + C_2 \, Sin \, M_2 x \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (31)$$

$$y = \frac{\lambda^2}{\theta_W V} + C_1 + C_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (32)$$

But if $x = \frac{v}{t}$

Therefore, equation (33) can be expressed as:

$$C(x,t) = \left(C_1 \, Cos \, M_1 \frac{\lambda^2}{\theta_W V} \frac{d}{v} + C_2 \, Sin \, M_2 \frac{\lambda^2}{\theta_W V} v.t \right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (33)$$

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2. Materials and method

Standard laboratory analysis to determine the deformation rate of concrete from magnesium sulfate were analyzed, this were done to monitor the rate of magnesium sulfate degrading concrete pavements formation magnesium concentration are discovered to deform concrete strength, the life span of concrete pavement are reduced when magnessium substances are found in concrete pavement, it will definitely reduce the compressive strength of concrete. Analysis were carried out, it is compare with theoretical values for model validation.

4. Results and Discussion

Results and discussion from the expressed figures through the theoretical generated values are presented in tables and figures, the expression explain the rate of concentration through graphical representation for every condition assessed in the developed model equations.

Table 1: Concentration of magnesium sulfate at Different Time

Time [Per Days]	Concentration[Mg/L]
3	84
6	168.77
9	253.15
12	337.54
15	421.92
18	506.31
21	590.04
24	675.08
27	759.47
30	843.56

Table 2: Concentration of magnesium sulfate at Different Time

Time [Per Day]	Concentration[Mg/L]
10	**84**
20	**168.77**
30	**253.15**
40	**337.54**
50	**421.92**
60	**506.31**
70	**590.04**
80	**675.08**

90	**759.47**
100	**843.56**

Table 3: Comparison of Theoretical and Experimental Values of magnesium sulfate concentration at Different time

Time [per Day]	Theoretical values [Mg/l]	Experimental values [Mg/L]
3	84	85
6	168.77	166.78
9	253.15	266.23
12	337.54	334.23
15	421.92	444.11
18	506.31	499.87
21	590.04	588.78
24	675.08	689.11
27	759.47	766.45
30	843.56	855.23

Table 4: Comparison of Theoretical and Experimental Values of magnesium sulfate concentration at Different time

Time [Per Day]	Theoretical values [Mg/l]	Experimental values [Mg/L]
10	84	85
20	168.77	166.78
30	253.15	266.23
40	337.54	334.23
50	421.92	444.11
60	506.31	499.87
70	590.04	588.78
80	675.08	689.11
90	759.47	766.45
100	843.56	855.23

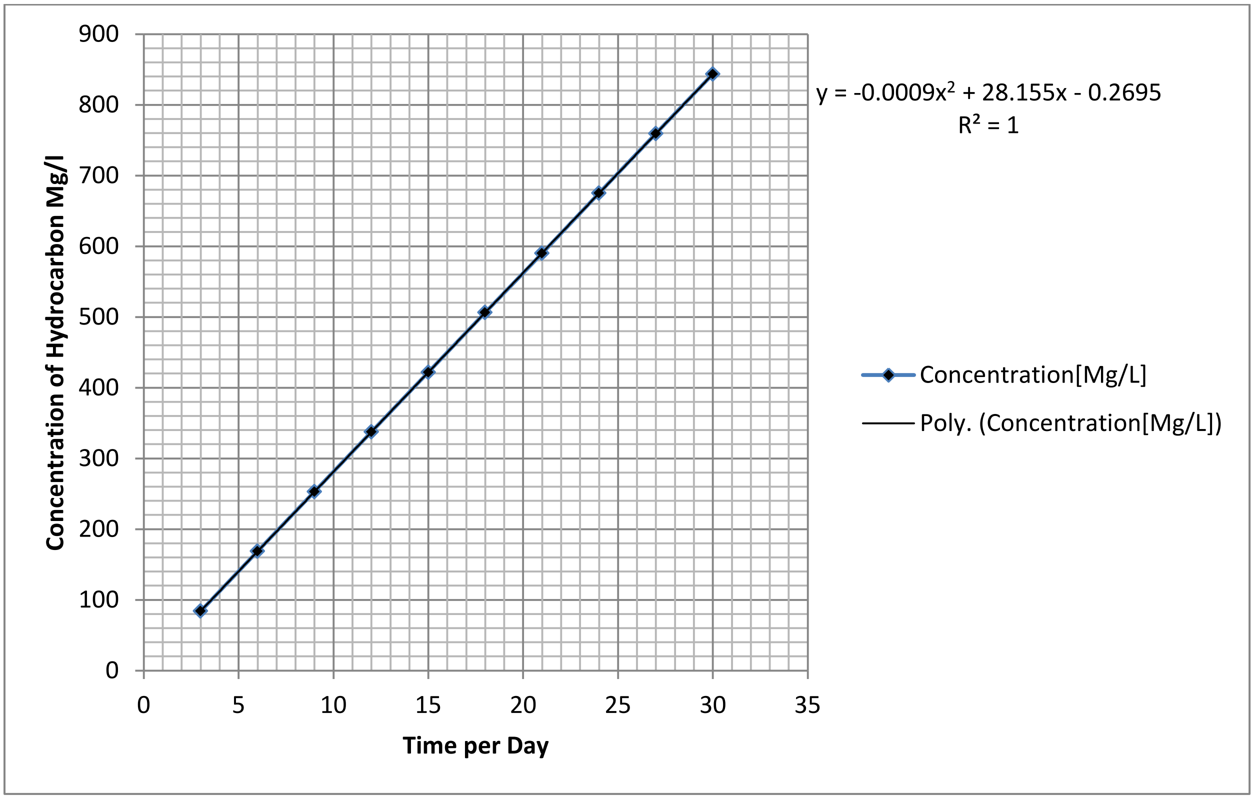

Figure 1: Concentration of magnesium sulfate at Different Time

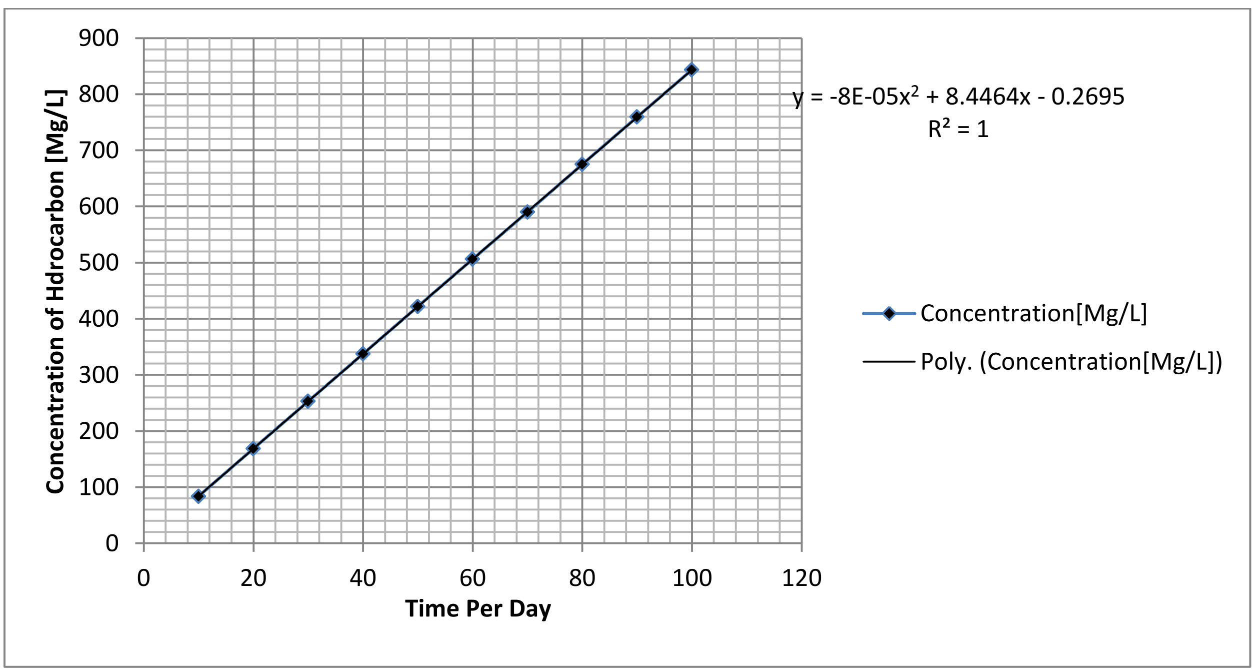

Figure 2: Concentration of magnesium sulfate at Different Time

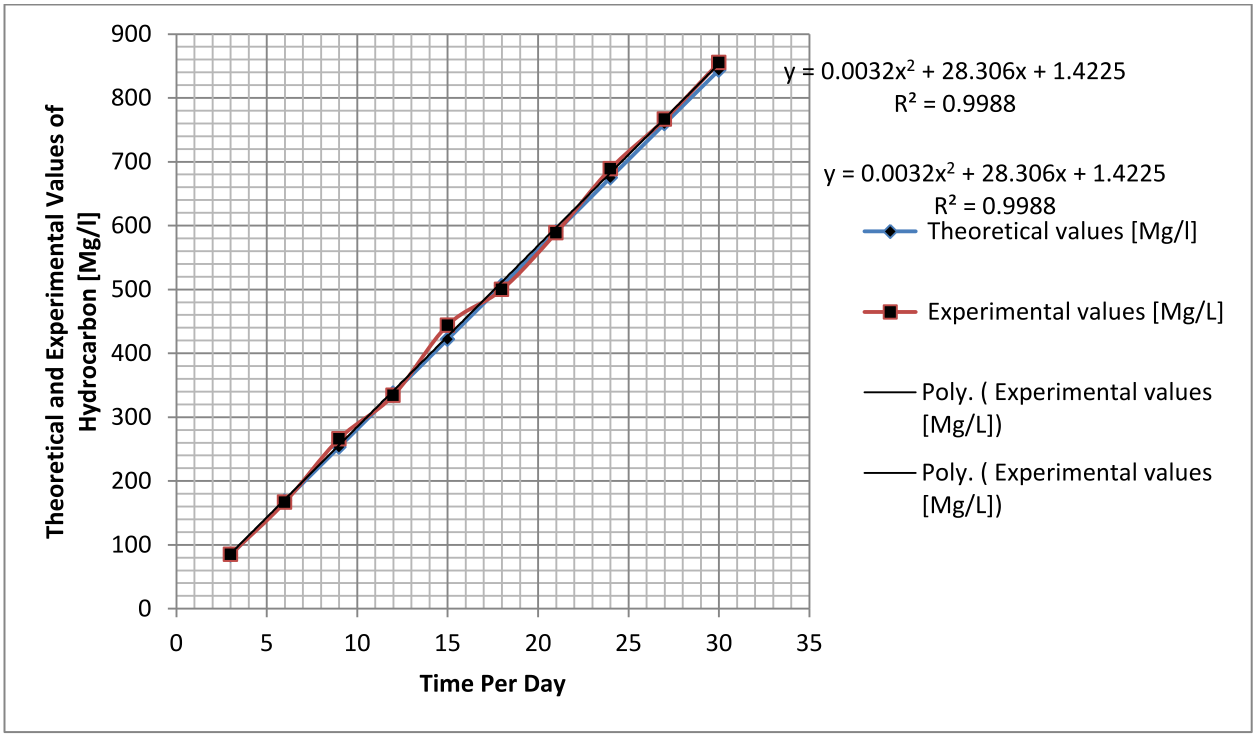

Figure 3: Comparison of Theoretical and Experimental Values chloride concentration at Different time

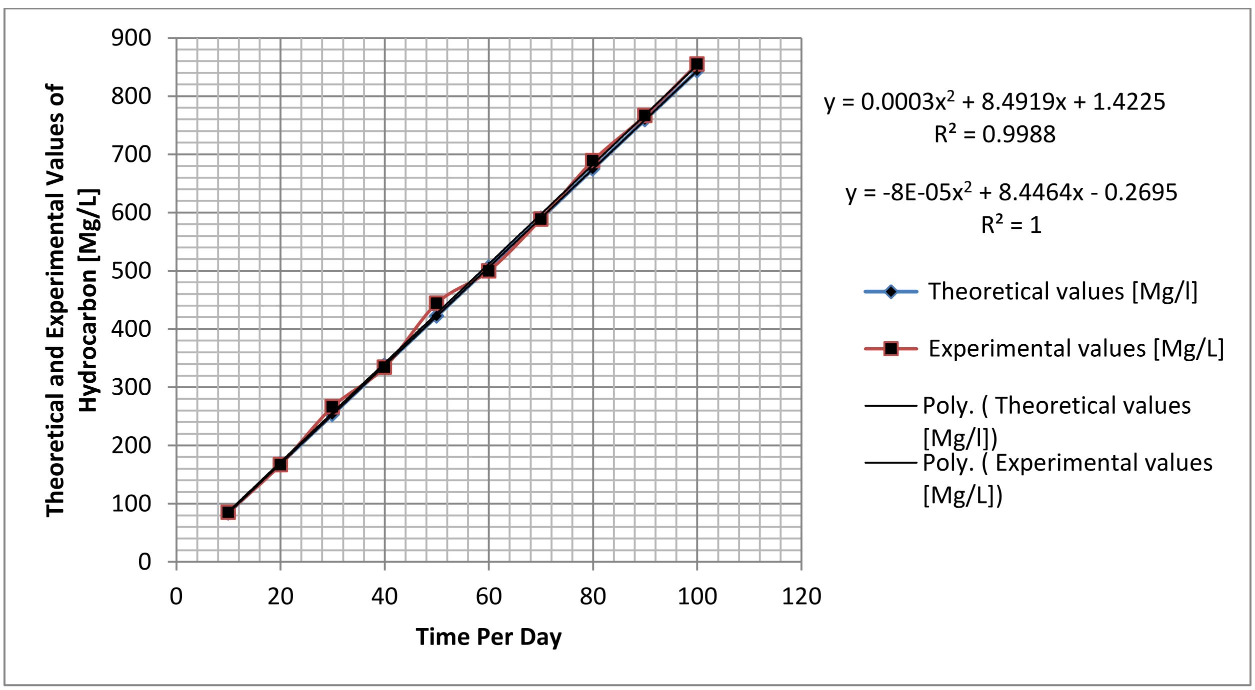

Figure 4: Comparison of Theoretical and Experimental Values chloride concentration at Different Time

The figure presented shows that the concentrations of magnesium sulfate in concrete pavement are in progressive phase, the rate of magnesium deposition in concrete pavement are determined from the constituent composition of the materials in concrete formation. The structure of concrete are determine by it characteristics. Therefore concrete formation varies base on its application, the formation characteristics of concrete pavement depends on the mix design, it also depend on the impose load that the pavement will carry, such condition the designed will be done to withstands its compressive strength from impose load that pass through the pavement, different trucks and other vehicles will move on the concrete pavement, increase on the magnesium sulfate reduces the concrete strength, the figure express the exponential rate of magnesium sulfate base on the increase in deformation pressured by the rate of magnesium sulfate, the expression from the figure presented shows the rate of damage caused by high concentration of magnesium sulfate in concrete pavement, such condition has cause lots of pavement failure when such precautions are not adhered in designed and construction of pavements, the deposition of magnesium sulfate in concrete formation should taken as a serious concerned since it reduce the compressive strength of concrete pavement, the developed model is imperative because it has determined the rates of magnesium sulfate attack on concrete formation, exponential phase of magnesium has express it rate deformation in concrete life span, experts will apply this conceptual formation to determine the rate of magnesium sulfate deformation in concrete pavements

4. Conclusion

Magnesium has been discovered to be a major attack on concrete compressive strength, the lifespan design for concrete pavement may reduce if such attack is not noticed in concrete characteristics. Such development has been found to deform concrete pavement when it is not noted. Subject to this relation, degradation of concrete formation should be expressed through concrete testing of compressive strength and other analyses. This will definitely establish the quality control of concrete formation in order to attain the designed life strength of the pavement. Magnesium sulfate in this study has been observed in several pavement failures to have deformed the attained strength in different times based on magnesium deposition in concrete pavement. Deformation of this substance has developed lots of design failures from deposition of magnesium sulfate. Based on these conditions, mathematical modeling and simulation were found suitable to monitor the rate of deformation through the deposition of magnesium sulfate in concrete pavement. Magnesium compounds are potentially more destructive to concrete as the Magnesium ion is capable of completely replacing the calcium in hydrated Portland cement. Magnesium sulfate is highly soluble; it can form more highly concentrated solid solutions than sodium sulfate. Strong solutions of Magnesium sulfate are capable of reacting with calcium silicate hydrate phases as well as calcium hydroxide and calcium aluminates hydrate phases. The concentration of magnesium sulfate may affect the course of the reaction.

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